Thoughts Behind the Actions: Exploring Preservice Teachers' Mathematical Content Knowledge

Leah Daniel
James Cook University
<leah.daniel@jcu.edu.au

Josephine Balatti
James Cook University
<josephine.balatti@jcu.edu.au>

Teacher educators need to identify those aspects of preservice teacher (PST) mathematical content knowledge (MCK) that need developing. A methodology that unpacks the MCK that PSTs use in their teaching is presented in this paper. MCK in the teaching acts themselves and in the PST reflections on those acts is categorised and evaluated. The process is illustrated with a lesson excerpt from a secondary mathematics PST. The benefits and limitations associated with this methodology are also discussed.

To "demonstrate knowledge and understanding of the concepts, substance and structure of the content" is part of the second Australian Professional Standard for Teachers "Know the content and how to teach it" required of the Australian Institute for Teaching and School Leadership (AITSL, 2012). To design effective opportunities for preservice teachers (PSTs) to develop their mathematical content knowledge (MCK), mathematics educators and researchers need ways of ascertaining MCK competence that are as comprehensive, accurate, and objective as possible for the intended purpose. The situated nature of the MCK enacted in the classroom and its sometimes hidden presence make developing methodologies that reveal and evaluate MCK a complex issue. This paper contributes a new approach to investigating PST MCK that we believe provides insights that other approaches may not. The approach unpacks the MCK both in the PST classroom actions and in the intents behind those actions and evaluates the MCK for shortcomings.

Mathematical Content Knowledge (MCK) for Teaching

When mathematics education researcher Deborah Ball and her colleagues (Ball & Bass, 2009; Ball, Thames & Phelps, 2008) reconceptualised Shulman's (1986) influential work on teacher knowledge through a mathematical lens, they highlighted the important place of MCK in teacher knowledge. They described MCK as a type of mathematical knowledge that can be known independently of students and teaching, but which, when enacted, shapes and is shaped by how teachers engage with the many elements that constitute the teaching context. Ball and Bass (2009) identify three dimensions of MCK: common content knowledge, specialised content knowledge and horizon knowledge. Specialised content knowledge includes explicit and unpacked knowledge of the principles that underpin mathematical facts, representations, language, and procedures. Horizon knowledge is the additional awareness of the "mathematical landscape in which the present experience and instruction is situated" (Ball & Bass, 2009, p. 6) and allows teachers to prepare their students more effectively for more advanced mathematical ideas. Common content knowledge provides the foundation for specialised and horizon knowledge and refers to knowledge that is commonly required in many mathematically demanding professions. In the approach described in this paper, common content knowledge is further deconstructed using Kilpatrick, Swafford and Findell's (2001) framework of mathematical proficiency. The framework comprises five proficiency strands of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and

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productive disposition. As productive disposition refers to the character of a person, for the purposes of this paper, only the first four proficiency strands are used.

Conceptual understanding includes the essential features and general principles underpinning concepts (Even, 1993; Skemp, 1978), broad concept definitions (Usiskin, 2001; Wu, 2008), and concept representations (Davis, 2008). Procedural fluency refers to the efficient, flexible, and appropriate use of mathematical procedures. The relationship between this strand and that of conceptual understanding is captured in Skemp's (1978) notion of relational understanding. The third strand, strategic competence, refers to the ability to think in different ways about mathematics and move flexibly between mathematical approaches to solving problems. Algebraic thinking, for example, would feature an approach using symbols and algebraic manipulation (Dindyal, 2007) whereas geometric thinking would feature the use of attributes and properties of figures (Burger & Shaughnessy, 1986). The fourth strand, adaptive reasoning refers to mathematical explanations and justifications based on reflection and logical thought. Over time, the MCK that teaching draws upon in action incorporates all of these strands.

Lave and Wenger's (1991) study of situated cognition highlights the unique nature of knowledge in action and the need to view knowledge as dynamic and located within a community of practice. Measuring PSTs' MCK using written tests and interviews (Ball, 1990; Bryan, 1999; Even, 1993) fails to account for the situated nature of MCK in the community of practice comprising teacher and students. There have been studies that have attempted to research PSTs' MCK in the classroom; Rowland, Huckstep, and Thwaites (2005), for example, developed the Knowledge Quartet framework to describe the actions of primary preservice teachers teaching mathematics and have found success modifying the framework for secondary settings (Thwaites, Jared & Rowland, 2011).

To study the PST's MCK only in how it manifests in the teaching act has its own limitations. The PSTs' competence in the other knowledges that Shulman (1986) identifies, such as pedagogical content knowledge and curricular knowledge, and their skill in applying them in a particular context can impact on the MCK that is evident in their teaching. To compensate for this limitation, we can turn to the intended purpose that the PSTs held for doing what they did in the classroom as an additional source of information.

Schoenfeld's (1999) Teaching-in-Context theory highlights the connection of a teaching act to a goal or intent and proposes that the intents are directly related to the knowledge and beliefs of that teacher. The actions of a teacher are the result of a decision made either before the lesson begins or in the moment of teaching and are performed with a goal in mind. In one study by Borko et al. (2000) that investigated teacher actions and the thoughts behind those actions, the intents that accompanied the teacher' actions were identified as valuable data sources worthy of interrogation. The study of intents behind teaching actions therefore provides another lens through which to study MCK.

In the methodology presented in this paper, the sites in which MCK is sought are in the acts the PST performs in the classroom and in the PST's reflections upon those acts, in particular, in the stated intents for those acts. By "act", we mean something that the teacher does in the classroom, and by "intent", we mean the purpose or goal that the PST hoped to achieve by performing that act. The intent may be either explicitly expressed in class and/or stated in the reflection on the act. In the data pertaining to an act and its accompanying reflection, there may be one or more MCK statements.

Reflections upon teaching acts have successfully been elicited using Lyle's (2003) stimulated recall techniques. Stimulated recall is a process where recorded footage of an event is replayed to a participant and they are invited, with minimal prompting, to recall the events and their thoughts at that time. The recalled commentaries offer researchers the opportunity to more accurately describe and more confidently interpret observed events by hearing the participant's own interpretations. It is perhaps for this reason that stimulated recall methods are considered the "least intrusive but most inclusive way of studying classroom phenomena" (Reitano & Sim, 2010, p. 218).

Evaluating the quality of PSTs' MCK enacted in the classroom is challenging given the many contextual factors that can impact the MCK manifested. The subjectivity of the evaluator can also impact the quality of the judgements made, with arguably qualitative judgements to do with excellence attracting more controversy than those to do with weaknesses or flaws. Correctness and incorrectness of MCK has been used as one measure of evaluation. Bryan (1999) found that PSTs can possess mathematics which is untrue. Even (1993) found preservice teachers' concept images to be inadequate or lacking particular features and Davis (2008) explains that when an inadequate image is extended beyond a very limited context its usefulness may fail. Adequacy of the mathematics could therefore be used to consider how well knowledge presented in one lesson may potentially extend into later ones. Hill and Charalambous (2012) note that PSTs with weak knowledge can be imprecise or careless with their representation of mathematical ideas, which indicates that precision may be another measure that can be used. Finally, Ball, Thames, and Phelps (2008) identify the possible presence of compressed knowledge, suggesting that "teachers need to not only be able to do mathematics but they need to unpack the elements of that mathematics to make its features apparent to students" (p. 10). Hence, the measures of correctness, adequacy, precision, and the extent to which the mathematics is suitably decompressed may be used as a set of criteria to evaluate the MCK evident in the teaching act and in the reflection, especially the intent(s) relevant to that act.

Elements of the Approach

The data that this approach requires is video footage of a teaching episode by a secondary PST. The footage provides observational data for the researcher and the stimulus for the PST to reflect on their actions in the stimulated recall interviews that take place shortly after the delivery of the lessons. Field notes that record contextual details that cannot be adequately captured with a video camera supplement the video footage.

Within 48 hours of the lesson observation, the PST reviews the edited footage of several lesson excerpts with a researcher using stimulated recall (Lyle, 2003). The data generated from the PST using this approach may include a) justifications such as intents, beliefs, and contextual factors for the decision to take a particular action; b) observations about the act itself; c) recollections of their thinking at the time the acts were taking place; and d) post-act reflections concerning, for example, the perceived effectiveness of the acts.

The analysis process comprising three phases begins with editing the footage to be used in the stimulated recall interviews.

Analysis phase one: Selecting teaching acts. Selecting the acts for which the MCK is analysed involves a two-step process. In the first step, the researcher selects from the raw video footage those acts where an explicit reference to mathematical content is made either verbally and/or in written form. In so doing, acts such as marking of a roll or managing behaviour are not included in the list but acts such as providing a mathematical explanation are included. The raw footage is then edited and reduced to those lesson excerpts that contain a higher proportion of MCK related acts than other excerpts, and/or excerpts that contain an MCK related act of particular interest to the researcher. In the interest of time constraints, the researcher uses discretion to determine how much footage of a particular act and its context is included in the video. The second step requires the PST to view the edited footage of the selected acts. The acts that attract comment from the PST either spontaneously or in response to the researcher's prompt provide the data sources for the second analysis phase. Significantly, the viewing of the edited footage can lead the PST to note "acts of omission" i.e., actions that they had consciously decided not to perform. Given that the act of omission and the thoughts behind the act can potentially provide further insight into the PST's MCK, these acts of omission are also selected for analysis.

Analysis phase two: Identifying the intents behind the acts. In the PST's reflections while watching the video, the purpose or intent behind the actions selected in phase one is either explicitly stated or implied in comments made about a specific act. Prompts are rarely needed for PSTs to articulate their intents. At times, more than one intent is expressed for an action. Not all intents need relate to MCK, such as an intent to save time or to follow the class routine. Only intents where an explicit reference to mathematics is made are selected for further analysis. By the end of phase two, the data are reduced to those acts that attracted reflection and to reflection statements that contain MCK.

Analysis phase three: Unpacking and evaluating the MCK. The data pertaining to MCK is first categorised into either incorrect or correct MCK. Incorrect MCK is used to categorise any aspect of MCK expressed in the act or in the PST reflection which is mathematically untrue. One example of incorrect mathematics from a PST's reflection was the following statement: "Uh, we use BOMDAS; so it's multiplication then division." Providing that there is sufficient evidence to make the judgement, the mathematics categorised as correct is further categorised using four of the proficiency strands of Kilpatrick et al. (2001). The strand, Strategic Competence, is further coded for the 'ways of thinking' evident in the data. At times, an act, intent or other reflection does not offer enough information to the researcher to categorise the MCK. For example, an open question such as "What do w e know about gradient?" lacks explicit reference to a particular proficiency strand. Keeping these constraints in mind, the MCK in an act, intent, or other PST comment can only be categorised if enough evidence exists to justify an unambiguous categorisation. The final stage of unpacking MCK is to review the acts, intents, and other PSTs comments for evidence of specialised or horizon knowledge.

The correct MCK is finally analysed for shortcomings. The three categories used are taken from the literature discussed earlier and are: *inadequate* – an aspect of MCK is noticeably absent; *imprecise* – MCK is used carelessly and lacks the necessary detail to be considered completely accurate (but is not incorrect), and *compressed* – the MCK is packed so tightly together that some aspects are inaccessible to the PST.

Illustration of the Process

The case chosen to illustrate the method is a seven minute excerpt from a Year 8 lesson taught by Ellen to a well-behaved class of 25 boys. Ellen is a 23 year old PST in her third year of a four year Bachelor of Education degree. Ellen's school based teacher educator had given her the task of reviewing student prior knowledge on the topic of area before teaching the area of composite shapes.

Area is the quantity of a region bounded by the contour of a figure (Berenson et al., 1997). This geometric property can be measured through estimation or calculation of the number of unit areas (conventionally square) that cover a region with no overlap (Battista, 1982), reflecting a geometric treatment of the area concept. Area can also be considered from an algebraic perspective, using algorithms to expedite the process of ascertaining square units to measure the area (Zacharos, 2006).

The excerpt is a whole of class discussion that took place after the students had individually responded in writing to three review questions that Ellen had written on the board (Figure 1). The day after the lesson, Ellen participated in a stimulated recall session with the researcher and the discussion pertaining to this excerpt was 23 minutes in length.

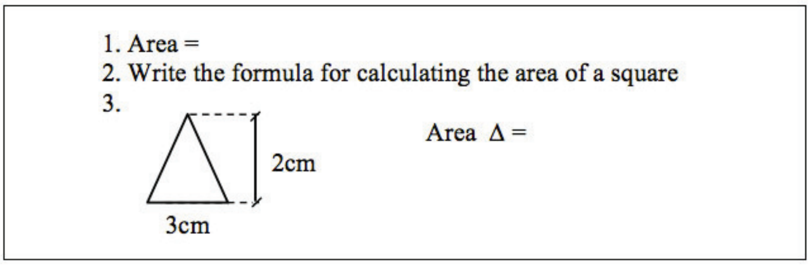

Figure 1. Review questions.

Analysis phase one: Selecting teaching acts. The researchers included 27 acts in the video footage used for the stimulated recall. Of these, Ellen commented on 14. For example, one act that drew comment was when she pointed to the first review question on the whiteboard and asked the class, "What is area?" In this case, the reflection only referred to the purpose in choosing the question (discussed in the next section) and did not include any reflection on what occurred in the classroom.

The stimulated recall also revealed "acts of omission" when Ellen had made the decision not to act. For example, she explained that she was uncertain how to indicate the "perpendicular" property of the height in the notation used for the formula for area of a triangle. She decided, based on this uncertainty, to avoid including any reference to the property, and instead used the letter "h" to represent perpendicular height: "I didn't write it up there 'cause I don't know what the abbreviation for perpendicular height is. Do you just write height or do you write p height?" With the inclusion of the "acts of omission" data a total of 18 acts were further analysed.

Analysis phase two: Identifying the intents behind the acts. Of the 18 acts discussed, Ellen's reflections revealed 23 intents for acting as she did. For this excerpt, all the intents implicated MCK directly or indirectly. Using the act introduced above as an example, Ellen explained her intent for posing the question "What is area?" as follows:

> The first question's just like an open answer. They could write 'length times width'; they could write 'side squared'; it could be like 'area of a square is this, area of a rectangle is that'. They could write 'the area is the occupied space in between…'[trails off]. I was happy with any of it …'cause, I wanted to see if they understood area.

Analysis phase three: Unpacking and evaluating the MCK. The data revealed *incorrect* MCK pertaining to two of the 18 acts with the incorrect MCK being evident only in Ellen's reflection on those two acts and not in her actual teaching. Table 1 summarises the unpacking of the *correct* MCK into categories. It should be noted that no correct MCK was coded under "Adaptive reasoning" as no significant evidence was found of Ellen reasoning beyond mathematical justifications such as, "that's the way it is."

Table 1
Categories of MCK pertaining to the 18 acts in the lesson excerpt

Source of evidence	Mathematical Content Knowledge					
	Common Content Knowledge				Specialised Content Knowledge	Horizon Knowledge
	Conceptual understanding	Procedural fluency	Strategic competence: Algebraic	Strategic competence: Geometric		
Act only	0/18 acts	0/18 acts	0/18 acts	0/18 acts	1/18 acts	0/18 acts
Reflection only	3/18 acts	3/18 acts	3/18 acts	3/18 acts	1/18 acts	1/18 acts
Act and reflection	1/18 acts	14/18 acts	13/18 acts	4/18 acts	4/18 acts	0/18 acts
Total	4/18 acts	17/18 acts	16/18 acts	7/18 acts	6/18 acts	1/18 acts

As an example, in Ellen's act of asking the class, "What is area?" only common content knowledge data were present. This is indicated with the four shaded boxes in the table above. The comment in Ellen's reflection in which she explained area as being "the occupied space in between" revealed an awareness of a conceptual notion of area and a geometric treatment of the topic. It was thus coded under 'Conceptual understanding' and 'Strategic competence: Geometric way of thinking'. It should be noted that neither of these were evident in the act itself. In contrast, 'Procedural fluency' and 'Strategic competence: Algebraic way of thinking' were evident in both the reflection and the act. The notation, "Area =" that Ellen pointed to during the act and her reference to area as "length times width" and "side squared" in the reflection revealed that she knew of procedures involving algorithms which could measure area. Ellen's question, "What is area?" provided insufficient evidence to categorise the mathematical knowledge involved.

In addition to the two acts that generated incorrect MCK, shortcomings in the correct MCK were evident in every act and are categorised in Table 2. Examples from the 18 acts in the excerpt illustrating each type of shortcoming are: *Inadequate*: unable to express area from a geometric point of view, *Imprecise*: drawing a diagram for review question three that lacked a right angle symbol in the height representation, and *Compressed*: unable to identify the source of student confusion because she was unaware of her own use of an equivalent symbol to represent division (a vinculum instead of an obelus). Shortcomings to do with incorrect, compressed, and inadequate MCK were found predominantly within Ellen's reflection whereas imprecise MCK appeared in both the acts and the reflection.

Table 2
Frequency of MCK shortcomings pertaining to the 18 acts in the lesson excerpt

Source of evidence	MCK Shortcoming			
	Incorrect MCK	Inadequate MCK	Imprecise MCK	Compressed MCK
Act only	0/18 acts	0/18 acts	4/18 acts	0/18 acts
Reflection only	2/18 acts	6/18 acts	2/18 acts	4/18 acts
Act and reflection	0/18 acts	1/18 acts	5/18 acts	1/18 acts
Total	2/18 acts	7/18 acts	11/18 acts	5/18 acts

The analysis of this seven minute excerpt alone suggests that Ellen's MCK regarding the topic of area could be improved. Even though the concept of area is located in the broader fields of measurement and geometry (Zacharos, 2006), this analysis suggests that Ellen has a preference for an algebraic treatment of the concept at the expense of a geometric one, basing her understanding of area on the use of algorithms. This propensity for formulaic approaches to area has been identified in previous research (Berenson et al., 1997; Bryan, 1999). A possible explanation for Ellen's emphasis of algebraic approaches may be her exposure to predominantly algebraic ways of thinking while undertaking advanced mathematics studies at a tertiary level. Nevertheless, this finding would suggest that a geometric treatment of the concept of area is beyond the scope of Ellen's MCK, resulting in an impoverished review.

Discussion and Conclusion

While a full picture of a PST's MCK is not possible, the illustration suggests that the methodological combination of classroom observation and stimulated recall interview data analysed with respect to MCK has the potential to provide more of the picture for a particular topic and context. Analysing both acts and reflections with a focus on intent can provide a better measure of the MCK that a PST holds, including the proficiency strands of common content knowledge, that influence teaching practice. This approach also allows the quality of MCK to be assessed and shortcomings in MCK quality identified.

Possibilities for further investigation include the analysis of all justifications, reflections and beliefs articulated about an act by the PST. It is envisaged that these additions would provide further insights regarding MCK. Using this approach to investigate the MCK that shapes intents which are achieved in the classroom and those intents where the actions do not allow the intent to be achieved offers additional opportunities for research. While this is beyond the scope of this paper, there were instances where Ellen's intent to develop her students' conceptual understanding was not matched by her classroom actions, inviting consideration of the influence of her MCK upon her intent and actions.

Logistical difficulties and limitations regarding this methodological process involve both the data collection and analysis stages. Logistical issues concern gaining access to PSTs, schools and particular classes. The approach is also time consuming due to the lengthy nature of tasks such as reviewing and editing the raw video footage and analysing the edited video footage and accompanying interview transcript. Limiting the selection of acts to those which have associated reflection is needed to reduce the potential for errors in analysis, but this means that some acts of interest to the researcher will be omitted. In addition, judgements about all aspects of MCK cannot be made for every act, as inferences about MCK can only be included if there is strong evidence to support the inference. These difficulties reflect the complexity of analysing knowledge which is implicit within a situated act (Borko et al., 2000) and concessions must inevitably be made in order to collect and analyse knowledge in action. Finally, the focus on shortcomings in the evaluation of the quality of the MCK may be considered as taking a deficit model approach. To this we offer the pragmatic response that it is through identifying MCK weaknesses that we can more effectively address them in PST education programs. We would argue that the potential to ascertain more accurately PSTs' MCK by analysing the teacher act and the thought behind the act outweighs the difficulties with the process, providing us with new opportunities to better understand the MCK of PSTs in practice.

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